Period-doubling route to chaos in a semiconductor laser subject to optical injection

T. B. SimpsonJ. M. LiuA. Gavrielides, V. Kovanis, and P. M. Alsing

Citation: **64**, 3539 (1994); doi: 10.1063/1.111218
View online: http://dx.doi.org/10.1063/1.111218
View Table of Contents: http://aip.scitation.org/toc/apl/64/26
Published by the American Institute of Physics

Period-doubling route to chaos in a semiconductor laser subject to optical injection

T. B. Simpson
JAYCOR, P.O. Box 84154, San Diego, California 92186-5154

J. M. Liu
Department of Electrical Engineering, University of California Los Angeles, Los Angeles, California 90024-159410

A. Gavrielides, V. Kovanis, and P. M. Alsing
Nonlinear Optics Center, Phillips Laboratory, 3350 Aberdeen Avenue SE, Kirtland AFB, New Mexico 87117-5776

(Received 4 January 1994; accepted for publication 13 April 1994)

Experimental measurements and a single-mode analysis of a quantum-well laser diode subject to strong optical injection are combined to demonstrate that the diode follows a period-doubling route to chaos. All laser parameters used in this model, including the influence of spontaneous emission noise, were experimentally determined based on the four-wave mixing technique. The transition to chaos can be used to reduce the uncertainty in the value of the linewidth enhancement factor.

In a variety of laser systems, a set of coupled equations for the complex oscillating field and the inversion density describes the nonlinear dynamics. An isolated, single-mode laser of this type, such as a semiconductor laser, is sufficiently described by only two rate equations; one for the photon density and another for the carrier density.[1,2] In order to induce chaotic dynamics in these systems, a third degree of freedom is needed, such as pump modulation or external optical injection.[1] Early theoretical and experimental investigations of lasers under external optical injection showed a variety of unstable dynamic phenomena, but did not consider a range of parameters relevant to semiconductor laser diodes.[1,3–5] More recently, it has been predicted that external optical injection into a laser diode can lead to chaos through the period-doubling route, though the experimental evidence has been ambiguous.[6]

In this letter, we present experimental evidence that strong optical injection into a laser diode from a master laser operating at the free-running oscillation frequency of the laser diode is sufficient to drive it into a chaotic regime through a period-doubling route. Experimentally observed features are demonstrated using a single-mode injection model[7] which includes spontaneous emission noise.[8] All parameters required for the model have been experimentally determined using the four-wave mixing technique,[9] allowing a quantitative comparison between experimentally measured spectra and the predictions of the model. We have found that the transition to chaos can be used to reduce the uncertainty in the measured value of the linewidth enhancement factor of the laser diode.

The lasers used were commercially available, nearly single-mode GaAs/AlGaAs quantum-well lasers, SDL model 5301-G1. Both the master and slave lasers were temperature and current stabilized. The output of the master laser was injected directly into the slave laser with careful alignment to have good coupling into the laser mode. Isolaters were used to make sure that no light was injected into the master laser. The optical spectrum of the output of the slave laser was monitored with a Newport SR-240C scanning Fabry–Perot which has a free spectral range of 2 THz and a finesse of greater than 50 000.

The master laser was tuned to the free-running oscillation frequency of the slave laser. Residual offsets, due to frequency jitter between the two lasers, were less than 100 MHz. The slave laser is injection locked, but the dynamics are not necessarily stable under such injection.[10] The master laser's central linewidth, <10 MHz, was significantly less than the linewidth of the free-running slave laser. Further, at the injection levels considered here, its noise spectra, away from the central peak, was significantly weaker than the spontaneous emission by the slave laser into its oscillating mode.

To determine the injection level, we calibrated the system by measuring the four-wave mixing spectrum in the weak injection limit and used our previously developed model of the interaction to compare the generated sideband signal with the central peak.[7,9] The normalized injection level is $\xi=(\eta|A_i|)/(\gamma_c|A_0|)$, where η is the coupling rate of the injected field, γ_c is the photon decay rate, and $|A_i|$ and $|A_0|$ are the injected and free-running oscillating field amplitudes, respectively. The injected power is proportional to ξ^2. Figure 1 shows two typical spectra of the principal mode of the slave laser. The frequency is relative to the free-running frequency or, equivalently, the injection frequency. In Fig. 1(a) $\xi=1.7\times10^{-2}$ and in Fig. 1(d) $\xi=2.6\times10^{-2}$. When taking these spectra, the slave laser was operated at an output level of 9 mW, where its free-running relaxation resonance frequency is $f_r=2.9$ GHz. When the optical injection is increased beyond the weak injection limit for stable injection looking, it first causes unstable oscillation of the laser at the resonance frequency. The relatively narrow peaks in the spectrum of Fig. 1(a) are separated by a frequency spacing of f_r and are typical features of highly unstable injection locking.[10] For injection levels, $\xi\lesssim1.2\times10^{-2}$, the optical spectrum contains very little output except in these narrow peaks. As the injection level is further increased, broad fea-

 0003-6951/94/64(26)/3539/3/$6.00

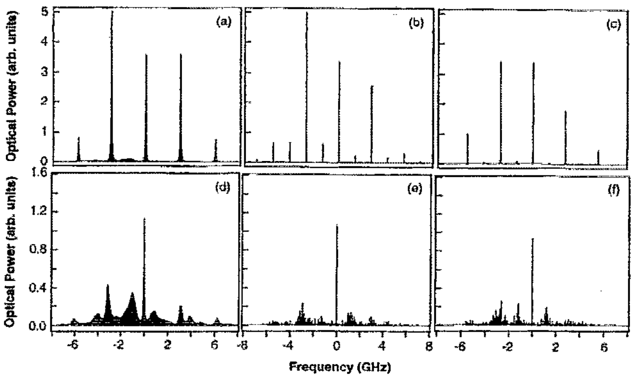

FIG. 1. Optical spectra of a semiconductor laser under optical injection at two levels in a period-doubling route to chaos: (a)–(c) $\xi=1.7\times10^{-2}$ and (d)–(f) $\xi=2.6\times10^{-2}$. (a) and (d) are experimentally measured spectra, (b) and (e) are numerically calculated spectra with no noise sources present, and (c) and (f) are calculated spectra with noise sources present. The parameters used in the calculations were experimentally determined using the four-wave mixing technique. Shading under the curves is a visual aid only.

tures appear in the spectrum between the narrow oscillation peaks, as is seen in Fig. 1(a). At even higher injection levels, the spectrum becomes dominated by a broad pedestal and many strong secondary peaks develop, as the spectrum in Fig. 1(d) shows. At this stage, chaos is fully developed, as will be demonstrated by comparison with calculated spectra. We have also observed that as the laser is driven into the chaotic region, the average oscillating power in the principal mode decreases by as much as 36% while the overall output is approximately constant. The power is spread among several of the weak side modes.

Our analysis employed the single-mode model which couples the complex oscillating field with the carrier density:[7]

$$\frac{dA}{dt}=-\frac{\gamma_c}{2}A+i(\omega_0-\omega_c)A+\frac{\Gamma}{2}(1-ib)gA + \eta A_i e^{-i\Omega t}+F_{sp}, \tag{1}$$

$$\frac{dN}{dt}=\frac{J}{ed}-\frac{N}{\tau_s}-\frac{2\epsilon_0 n^2}{\hbar\omega_0}g|A|^2. \tag{2}$$

Here, A is the total complex intracavity field amplitude at the free-oscillating frequency ω_0, ω_c is the longitudinal-mode frequency of the cold laser cavity, Γ is the confinement factor, b is the linewidth enhancement factor, g is the gain coefficient that includes second-order effects, A_i is the complex amplitude of the injection signal, and Ω is the detuning of the injection signal, F_{sp} is the spontaneous emission Langevin noise source and is assumed to have a correlation time short compared to γ_c^{-1},[2] N is the carrier density, J is the injection current density, e is the electronic charge, d is the active layer thickness, τ_s is the carrier lifetime, and n is the refractive index of the semiconductor medium. In order to perform numerical simulations using the coupled-equation model, we have transformed Eqs. (1) and (2) into a set of three real equations in a form which shows explicitly the dependence on the laser parameters experimentally determined in the weak injection limit:

$$\frac{da}{dt}=\frac{1}{2}\left(\frac{\gamma_c\gamma_n}{\gamma_s\tilde{J}}\tilde{n}-\gamma_p(2a+a^2)\right)(1+a) + \eta a_i\cos(\Omega t+\phi)+F'/|A_0|, \tag{3}$$

$$\frac{d\phi}{dt}=-\frac{b}{2}\left(\frac{\gamma_c\gamma_n}{\gamma_s\tilde{J}}\tilde{n}-\gamma_p(2a+a^2)\right) - \frac{\eta a_i\sin(\Omega t+\phi)-F''/|A_0|}{1+a}, \tag{4}$$

$$\frac{d\tilde{n}}{dt}=-\gamma_s\tilde{n}-\gamma_n(1+a)^2\tilde{n}-\gamma_s\tilde{J}(2a+a^2) +(\gamma_s\gamma_p/\gamma_c)\tilde{J}(2a+a^2)(1+a)^2, \tag{5}$$

where $a=(|A|-|A_0|)/|A_0|$, $a_i=|A_i|/|A_0|$, ϕ is the phase difference between A and A_i, $\tilde{n}=(N-N_0)/N_0$, $\tilde{J}=(J/ed-\gamma_s N_0)/\gamma_s N_0$, $\gamma_s=1/\tau_s$, N_0 is the steady-state, free-running carrier density, and γ_n and γ_p are relaxation rates contributed by the differential and nonlinear gain parameters, respectively.[7,9] We have made use of the stochastic nature of F_{sp} and its short correlation time to generate the two Langevin source terms, F' and F'' in Eqs. (3) and (4).

The coupled equations were numerically integrated, and the resulting times series were Fourier transformed, for various injection levels. Noise and dynamic parameters of the laser were independently determined by the four-wave mixing experiment,[8,9] and, in the results presented here, $\Omega=0$, to model optical injection at the free-running frequency. Two sets of calculated optical spectra were obtained at injection levels similar to the corresponding experimental spectra shown in Fig. 1. The spectra in Figs. 1(b) and 1(e) are calculated with no spontaneous emission noise term in the field equations, while the spectra in Figs. 1(c) and 1(f) include a spontaneous emission noise source. The positions of the computed peaks appear at the same frequencies as the corresponding experimental peaks. Relative strengths of the computed peaks are consistent with the experimentally obtained spectra when noise is present, though there is some discrepancy in the details of the different peaks.

The period-doubling features are most evident in the deterministic spectra, Fig. 1(b). A principal difference between the two sets of calculated spectra is that the noise term leads to a broadening of the period-doubling features in Fig. 1(c), consistent with the experimental data of Fig. 1(a). The noise would tend to obscure the identification of these peaks without our detailed work to determine the dynamic and noise parameters of the laser diode. Broadening of the relaxation oscillation side peaks is accompanied by a reduction in the height of the peaks. Fully developed chaos is observed in the spectrum shown in Fig. 1(e), where the broadening in the calculated optical spectra is achieved without any stochastic sources present in Eqs. (3) and (5).

Figure 2 depicts the numerically obtained bifurcation diagram of the values of the extrema of the electric field amplitude versus the injection parameter ξ when the noise source terms are not included. Both the initial bifurcation point and first period-doubling point occur at injection levels consistent with the experimental measurements. Only the initial instability region, first period-doubling region, and cha-

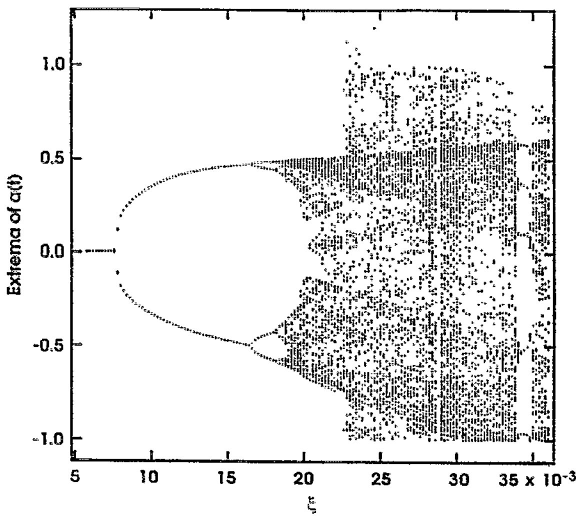

FIG. 2. Numerically obtained bifurcation diagram of the extrema of the normalized optical field amplitude, $a(t)$, vs the normalized injection level, ξ, showing the period-doubling route to chaos. Experimentally determined parameters of the laser are used in the calculation.

otic region exist over an experimentally distinguishable range of injection values because of the noise. One may view the effect of spontaneous emission noise as causing a fluctuating optical injection. Part of its influence is, effectively, to rapidly move the laser diode output to different injection levels on the bifurcation diagram.

It was also found that relatively minor changes in the linewidth enhancement factor can induce large changes in the calculated bifurcation diagram and optical spectra. The numerical results discussed above were obtained with a linewidth enhancement factor of $b=3.47$, which is well within the range of values, 3 ± 1 determined by the four-wave mixing experiment.[9] We have found that the laser would not develop into a chaotic state if $b\lesssim3.1$. The transition to chaos can be used to narrow the uncertainty in b to 3.6 ± 4.

The nonlinear dynamics of a laser diode under external optical injection is sensitive to a variety of laser parameters. Here, we have demonstrated the period-doubling route to chaos for optical injection at the free-running frequency of the laser diode. Detuning the injection frequency will change the nonlinear dynamics, and is predicted to lead to different routes to chaos for lasers which can be modeled by the coupled equations for the complex oscillating field and population inversion.[1,3,11] In the analysis discussed above, the effect of side modes was not included. The single-mode model used here cannot, of course, account for the partition of power to the side modes and further work is needed to quantify the effect of the side modes. The good agreement between the spectra calculated from the single-mode model with noise and the experimental data indicates that the high-frequency dynamics of the system is not dominated by the presence of the side modes for these experimental conditions.

The authors thank F. Doft for technical assistance. T. B. S. and J. M. L. were supported by the Air Force's Phillips Laboratory Contract No. F29601-91-C-0081. V. K. would like to thank the National Research Council for support through an NRC Associateship.

[1] J. R. Tredicce, F. T. Arecchi, G. L. Lippi, and G. P. Puccioni, J. Opt. Soc. Am. B **2**, 173 (1985).

[2] G. P. Agrawal and N. K. Dutta, *Long-Wavelength Semiconductor Lasers* (Van Nostrand Reinhold, New York, 1986), Chap. 6.

[3] D. K. Bandy, L. M. Narducci, and L. A. Lugiato, J. Opt. Soc. Am. B **2**, 148 (1985).

[4] K. Otsuka and H. Kawaguchi, Phys. Rev. A **29**, 2953 (1984).

[5] J. L. Boulnois, A. Van Lerberghe, P. Cottin, F. T. Arecchi, and G. P. Puccioni, Opt. Commun. **58**, 124 (1986).

[6] J. Sacher, D. Baums, P. Panknin, W. Elsasser, and E. O. Göbel, Phys. Rev. A **45**, 1893 (1992).

[7] T. B. Simpson and J. M. Liu, J. Appl. Phys. **73**, 2587 (1993), and J. M. Liu and T. B. Simpson, IEEE J. Quantum Electron. (to be published).

[8] T. B. Simpson, J. M. Liu, V. Kovanis, T. Gavrielides, and C. M. Clayton, Proc. SPIE **2039**, 23 (1993).

[9] J. M. Liu and T. B. Simpson, IEEE Photonics Technol. Lett. **4**, 380 (1993).

[10] I. Petitbon, P. Gallion, G. Debarge, and C. Chabran, IEEE J. Quantum Electron. **24**, 148 (1988).

[11] E.-K. Lee, H.-S. Pang, J.-D. Park, and H. Lee, Phys. Rev. A **47**, 736 (1993).